

# Constraints on Primordial non-Gaussianity from Future HI Intensity Mapping Experiments

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The primordial non-Gaussianity induces scale-dependent bias of the HI with respect to the underlying dark matter, which exhibits features on the very large scale of 21-cm power spectrum potentially observable with HI intensity mapping observations. We forecast the prospective constraints on the four fundamental shapes of primordial non-Gaussianity (local, equilateral, orthogonal and enfolded), with the current and future HI intensity mapping experiments, BINGO, FAST and SKA-I. With the current configuration of the experiments and assumed one-year observation time, we find that the SKA-I will provide tighter constraints on local shape of primordial non-Gaussianity than *Planck*. The results are  $(\sigma_{f_{\text{NL}}^{\text{local}}}, \sigma_{f_{\text{NL}}^{\text{equil}}}, \sigma_{f_{\text{NL}}^{\text{orth}}}, \sigma_{f_{\text{NL}}^{\text{enfold}}})_{\text{SKA-I}} = (0.54, 86, 25, 43)$ ,  $(\sigma_{f_{\text{NL}}^{\text{local}}}, \sigma_{f_{\text{NL}}^{\text{equil}}}, \sigma_{f_{\text{NL}}^{\text{orth}}}, \sigma_{f_{\text{NL}}^{\text{enfold}}})_{\text{BINGO}} = (17, 100, 128, 164)$ ,  $(\sigma_{f_{\text{NL}}^{\text{local}}}, \sigma_{f_{\text{NL}}^{\text{equil}}}, \sigma_{f_{\text{NL}}^{\text{orth}}}, \sigma_{f_{\text{NL}}^{\text{enfold}}})_{\text{FAST}} = (9.5, 44, 75, 94)$ . If the lower frequency band of FAST can be used, the constraint on local-type primordial non-Gaussianity will be  $\sigma_{f_{\text{NL}}} \sim 1.62$  which is better than *Planck*. In addition, if the observation time for FAST could be extended to two years, the constraint on equilateral shape of primordial non-Gaussianity will be improved to be  $\sigma_{f_{\text{NL}}} \sim 32$ . Similarly, if observational time of SKA-I could be extended to two years, the constraint on local and orthogonal shapes can be improved to 0.43 and 20 respectively, achieving better constraints than *Planck*.

## I. INTRODUCTION

The statistical properties of the primordial fluctuation offer rich insights into the physics of inflation and the early universe [1]. One of the wildly discussed questions is whether or not the primordial fluctuations are deviated from Gaussian distribution. The simple single-field slow-roll inflationary model predicts primordial fluctuation with almost Gaussian distribution [2]. However, many alternative models of single-field slow-roll inflation can produce different types of primordial non-Gaussianity [3–10] (hereafter PNG), which leaves distinctive features in statistical properties of cosmic microwave background (CMB) and the large-scale structure (LSS) of the Universe.

If the primordial fluctuation is Gaussian, the two-point correlation function (i.e. power spectrum in Fourier space) can describe all of the statistical properties of PNG. Therefore, the most straightforward way to measure the PNG is through the higher-order correlation of CMB or LSS. Current measurements of the temperature and polarization of CMB from *Planck* satellite provides the state-of-the-art constraints on local, equilateral and orthogonal types of PNG [11] as  $f_{\text{NL}}^{\text{local}} = 0.8 \pm 5.0$ ,  $f_{\text{NL}}^{\text{equil}} = -4 \pm 43$  and  $f_{\text{NL}}^{\text{ortho}} = -26 \pm 21$  at 68% confidence level (CL).

Besides the constraints from CMB, there have been a lot of efforts of measuring  $f_{\text{NL}}$  through large-scale structure surveys. This is because the PNG induces a scale-dependent bias of the galaxy with respect to the underlying dark matter distribution tracer [12–18]. Ref. [15] used spectroscopic and photometric luminous red galaxy samples, and quasars samples

from SDSS survey and obtain the limit for local type PNG as  $-31(-96) < f_{\text{NL}}^{\text{local}} < +70(+96)$  at 95% (99.7%) CL, which was comparable to the then measurements from *Wilkinson Microwave Anisotropy Probe* (WMAP) five-year results. Ref. [19] used radio sources from the NRAO VLA Sky Survey (NVSS), the quasar and MegaZ-LRG (DR7) catalogues of the SDSS, and the final SDSS II Luminous Red Galaxy (LRG) photometric redshift survey and found  $f_{\text{NL}}^{\text{local}} = 48 \pm 20$  ( $1\sigma$  CL). Ref. [20] found  $f_{\text{NL}}^{\text{local}} = 90 \pm 30$  at  $1\sigma$  CL by using photometric SDSS data, but due to unaccounted systematics this result may be better interpreted as  $f_{\text{NL}}^{\text{local}} < 120$  at 84 per cent CL. Ref. [21] used SDSS-III Baryon Oscillation Spectroscopic Survey (BOSS) data to constrain the  $f_{\text{NL}}^{\text{local}}$ , and found  $-45 < f_{\text{NL}}^{\text{local}} < 195$  at  $2\sigma$  CL. In addition, Ref. [22] used the correlation of the residual peculiar velocities on different directions to constrain PNG and found  $|f_{\text{NL}}^{\text{local}}| < 25.7$  at 68% CL. These limits are currently consistent with but weaker than the measurements from the *Planck* CMB observation. However, forecasts indicate that the constraint errors could decrease one or two orders of the magnitude with the future LSS survey, especially for the future radio survey. (see [23] and its references for review).

The scale-dependent bias not only affects the large-scale galaxy bias, but also affects the HI distribution. A more efficient method of the radio survey is to map out a large volume of the Universe through the intensity mapping technique, which measures the combined HI emission of the unresolved galaxies. Therefore, in principle one can obtain a three-dimensional HI distribution that can provide more modes of fluctuations than the CMB two-dimensional sphere. There have been several works to forecast the detectability of PNG through HI intensity mapping technique [24–26], but those forecasts are exclusively only for local and equilateral type of PNG and limited experimental cases (SKA and Tianlai). In this work, we will calculate the scale-dependent bias of all four typical types of PNG by using the halo model, and cal-

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culate their imprints on the power spectrum of HI. Then we forecast the detectability of all of the three on-going HI imaging surveys, i.e. BAO as Integrated Neutral Gas Observation (BINGO) [27], Five-Hundred-Metre Aperiodic Spherical Telescope (FAST) [28, 29] and Square Kilometre Array Phase-I (SKA-I) [30].

This paper is organized as follows. In Sect. II we summarize the primordial bispectrum and discuss different types of PNG to be forecasted in this work. In Sect. III we calculate the scale-dependent bias of the LSS induced by the PNG, and then the power spectrum of HI. In Sect. IV we introduce the Fisher matrix forecast method that used in our analysis. In Sect. V, the detailed experiment parameters are discussed. In Sect. VI we present our results and some discussion. Conclusion will be in the last section.

Besides the PNG parameters, we will adopt a spatially flat Universe with cosmological parameters fixed as *Planck* 2015 best-fitting values [31], i.e.  $\Omega_m = 0.309$ ;  $\Omega_\Lambda = 0.691$ ;  $\sigma_8 = 0.809$ ; and  $h = 0.68$ , where the Hubble constant is  $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$ . The amplitude and tilt of scalar power spectrum are  $A_s(k_0) = 2.141 \times 10^{-9}$  and  $n_s = 0.961$ , where pivot scale is  $k_0 = 0.002 \text{ Mpc}^{-1}$ .

## II. PRIMORDIAL BISPECTRUM

Most of the single-field inflationary models predict the primordial curvature fluctuations with the deviation from Gaussian distribution [32–34]. The deviation is particularly described by writing the gauge-invariant Bardeen’s potential  $\phi$  as the sum of a Gaussian random field and a quadratic correlation [33, 35],

$$\phi = \phi_G + f_{\text{NL}}(\phi_G^2 - \langle \phi_G^2 \rangle), \quad (1)$$

in which,  $f_{\text{NL}}$  is a dimensionless, phenomenological parameter describing the magnitude of the PNG.

To extract more information of the non-Gaussian primordial fluctuations, we need to go beyond the statistics of power spectrum. The lowest-order statistics sensitive to the PNG is the three-point function or bispectrum  $B_\phi(k_1, k_2, k_3)$ , in which,  $\phi$  is the primordial Bardeen potential which is directly related to the curvature perturbation [36]. The potential of primordial curvature perturbation is related to the Newtonian potential during the matter domination via the transfer function  $T(k)$  which satisfies  $T(k \rightarrow 0) = 1$ . By applying the Poisson equation,  $\phi$  is related to the matter density field  $\delta_m(k)$  by  $\delta_m(k) = \mathcal{M}(k)\phi(k)$ , where,

$$\mathcal{M}(k) = \frac{2}{3} \frac{k^2 T(k)}{\Omega_m H_0^2}. \quad (2)$$

The configuration shape of  $B_\phi(k_1, k_2, k_3)$  is related to the physical mechanisms during the inflation. In our analysis, we consider four classes of bispectrum shape characterizing the local, equilateral, enfolded and orthogonal types of PNG.

### A. Local shape

The standard single-field inflation predicts the local-type PNG [37]. But the parameter  $f_{\text{NL}}^{\text{local}}$  is expected to be very close to zero [2]. However, large amount of non-Gaussianity can be produced with different inflationary model, such as multifields model [3] or curvaton models [4]. In these cases,  $f_{\text{NL}}^{\text{local}}$  can be substantially different from zero.

The potential bispectrum of the local-type PNG has the simple form,

$$B_\phi(k_1, k_2, k_3) = 2f_{\text{NL}}^{\text{local}} [P_\phi(k_1)P_\phi(k_2) + (\text{cyc.})], \quad (3)$$

in which,  $P_\phi(k) = 2\pi^2 A_s(k_0)(k/k_0)^{n_s-4}$  is the power spectrum of the Gaussian Bardeen potential.

### B. Equilateral shape

The equilateral-type of PNG can be produced by the inflationary models with higher-derivative interactions. Usually there are two dominant interaction terms of the inflation field giving rise to the PNG. One of the dominant terms leads to the equilateral-type of PNG, and another leads to the orthogonal-type.

The primordial bispectrum of the equilateral type takes the form [6],

$$\begin{aligned} B_\phi(k_1, k_2, k_3) = & 6f_{\text{NL}}^{\text{equil}} \gamma(k_1, k_2, k_3) \\ & \times \left[ - \left( P_\phi(k_1)P_\phi(k_2) + (\text{cyc.}) \right) \right. \\ & - 2 \left( P_\phi(k_1)P_\phi(k_2)P_\phi(k_3) \right)^{2/3} \\ & \left. + \left( P_\phi^{1/3}(k_1)P_\phi^{2/3}(k_2)P_\phi(k_3) + (\text{cyc.}) \right) \right], \end{aligned} \quad (4)$$

in which, function  $\gamma(k_1, k_2, k_3)$  takes into account of the running of  $f_{\text{NL}}^{\text{equil}}$  and reads [38],

$$\gamma(k_1, k_2, k_3) = \left[ \frac{k_1 + k_2 + k_3}{k_{\text{CMB}}} \right]^{-2\kappa}, \quad (5)$$

where  $k_{\text{CMB}} = 0.086 h \text{ Mpc}^{-1}$  and  $\kappa = -0.2$ .

### C. Orthogonal shape

The orthogonal shape of PNG bispectrum is nearly orthogonal to both the local and equilateral forms of PNG, which is well approximated by the following template [39]

$$\begin{aligned} B_\phi(k_1, k_2, k_3) = & 6f_{\text{NL}}^{\text{orth}} \left[ - 3 \left( P_\phi(k_1)P_\phi(k_2) + (\text{cyc.}) \right) \right. \\ & - 8 \left( P_\phi(k_1)P_\phi(k_2)P_\phi(k_3) \right)^{2/3} \\ & \left. + 3 \left( P_\phi^{1/3}(k_1)P_\phi^{2/3}(k_2)P_\phi(k_3) + (\text{cyc.}) \right) \right], \end{aligned} \quad (6)$$

### D. Enfolded shape

It is well studied that if the initial vacuum state for the inflation deviates from the standard Bunch-Davies vacuum, the resulting bispectrum takes the enfolded-shape [7–10], which can be approximate by

$$B_\phi(k_1, k_2, k_3) = 6f_{\text{NL}}^{\text{enfold}} \left[ \left( P_\phi(k_1)P_\phi(k_2) + (\text{cyc.}) \right) + 3 \left( P_\phi(k_1)P_\phi(k_2)P_\phi(k_3) \right)^{2/3} - \left( P_\phi^{1/3}(k_1)P_\phi^{2/3}(k_2)P_\phi(k_3) + (\text{cyc.}) \right) \right]. \quad (7)$$

### III. HI BIAS AND POWER SPECTRA OF 21-CM

The HI bias is the bias of HI distribution with respect to the underlying dark matter distribution and the HI bias function,  $b_{\text{HI}}$ , can be obtained by assuming a model for the amount of HI mass in a dark matter halo of mass  $M$ ,  $M_{\text{HI}}(M)$ , and integrating over the halo mass function  $dn/dM$ . Here we use the Sheth-Tormen halo mass function [40] with mass range  $[10^8, 10^{16}]M_\odot$

$$b_{\text{HI}}(z) = \frac{1}{\rho_{\text{HI}}(z)} \int_{M_{\text{min}}}^{M_{\text{max}}} dM \frac{dn}{dM}(M, z) M_{\text{HI}}(M) b(M, z), \quad (8)$$

in which,  $b(M, z)$  is the real-space halo bias and  $\rho_{\text{HI}}(z)$  is,

$$\rho_{\text{HI}}(z) = \int_{M_{\text{min}}}^{M_{\text{max}}} dM \frac{dn}{dM}(M, z) M_{\text{HI}}(M). \quad (9)$$

For the HI intensity mapping experiments, we follow the assumption discussed in [41] and consider a simple power law model for the amount of HI mass,

$$M_{\text{HI}}(M) = AM^\alpha, \quad \alpha \simeq 0.6, \quad (10)$$

which is a redshift independent function. The pre-factor  $A$  will be canceled with the normalization of  $\rho_{\text{HI}}$ .

#### A. The Lagrangian bias

The Lagrangian bias describes the statistical bias of the halo distribution to the primordial dark matter fields. The PNG affects the initial conditions of the primordial density fields, so it is more convenient to study such effects in Lagrangian space. On the other hand, it is also necessarily to study the statistics of the evolved halo field at low redshifts in Eulerian space, which is conveniently related to the observation. The bias in Lagrangian space,  $b_L$ , relates to the Eulerian space bias,  $b_E$ , via  $b_E = b_L + 1$  [42]. The extra unity factor of  $b_E$  reflects the motions of primordial peaks at later times [22]. The uniformly distributed halos in the initial epoch, which have the  $b_L = 0$ , will lead to unbiased distribution to the dark matter field at later time. The  $b_L$  for halos is positive defined. But for

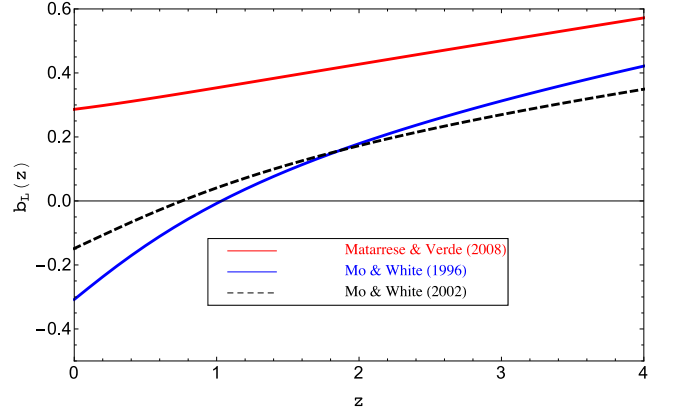


FIG. 1: Three models of Lagrangian bias  $b_L(z)$ , i.e. Matarrese and Verde [13], Mo and White [42], and Mo and White [43].

other dark matter tracers, it can be negative. The tracers anti-correlated with the initial dark matter fields will lead to the less clustered distribution than the dark matter field at later time.

It the past thirty years, people have been developing different analytical, semi-analytical and parametric models of the bias function. In below, we list the three most typical and commonly-used ones.

Basing on the Press & Schechter (hereafter PS) halo mass function [44] and its extensions, Mo and White (1996) [42] gives the bias factor for halos of mass  $M$ ,

$$b_L(M, z) = \frac{1}{\delta_c} [\nu^2(M, z) - 1], \quad (11)$$

where  $\nu(M, z) = \delta_c(z)/\sigma_R$ .  $\delta_c(z) = \delta_c/D(z)$ , where  $D(z)$  is the linear growth function which we use eq. (10) in [43] to compute it.  $\delta_c \simeq 1.686$  is the critical density contrast for spherical collapse. With the approximation of high-peak, the above bias factor can be expressed as  $b_L(M, z) = \delta_c(z)/\sigma_R^2$  (Matarrese and Verde 2008 [13]). With the ellipsoidal collapse model [45], Mo and White (2002) [43] gives another expression,

$$b_L(M, z) = \frac{1}{\delta_c(z)} \left[ \nu'^2 + b\nu'^{2(1-c)} - \frac{\nu'^{2c}/\sqrt{a}}{\nu'^{2c} + b(1-c)(1-c/2)} \right], \quad (12)$$

in which,  $\nu' = \sqrt{a}\nu$  and  $a = 0.707$ ,  $b = 0.5$ ,  $c = 0.6$ .

Figure 1 shows the three models of Lagrangian bias we discussed above.

#### B. The scale-dependent bias

As we analyzed before, PNG affects the distribution of the peaks at the initial stage of matter fluctuations, therefore it is correlated with the Lagrangian bias. In the presence of PNG, the halo bias can be written as the combination of a usual

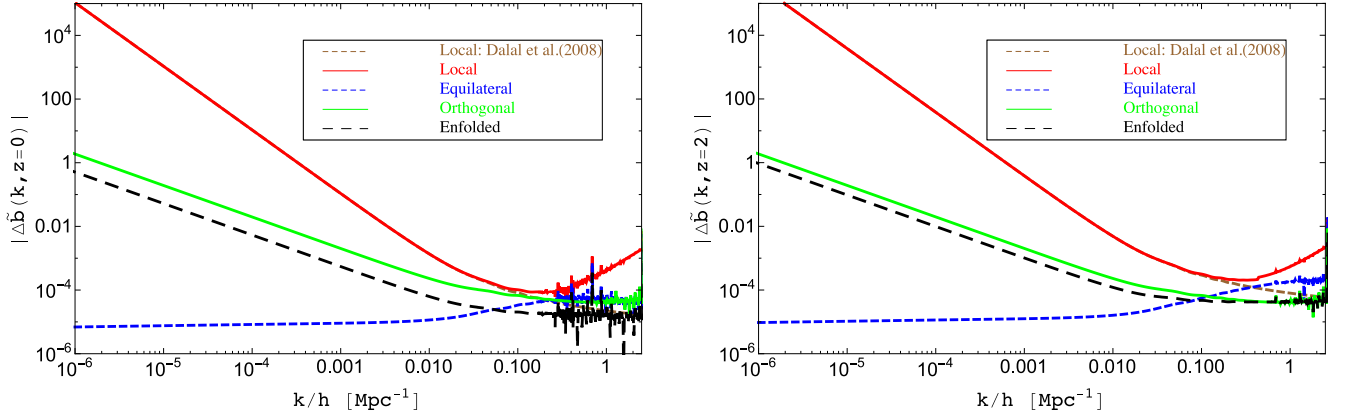


FIG. 2: The absolute value of scale-dependent bias  $|\Delta b(z, k)|$  (Eq. (15)) for different PNG shapes at  $z = 0$  (left panel) and  $z = 2$  (right panel) with assumed  $f_{\text{NL}} = 1$ . The four shapes of PNG are shown in different colour and dashed lines listed in the legend. The reason to plot absolute value is because orthogonal shape of  $\Delta b$  is negative (see also fig. 1 in [46]). The approximation of local shape of PNG by Dalal et al. [12] (Eq. (16)) is shown in the brown dashed line, which is consistent and almost completely overlapped with the computation from the halo model (Eqs. (3) and (17)) in red solid line.

scale-invariant bias,  $b(M, z)$ , and a scale-dependent modification,  $\Delta b(M, z, k)$ ,

$$b^{\text{NG}}(M, z, k) = b(M, z) + \Delta b(M, z, k). \quad (13)$$

By substituting Eq. (13) into Eq. (8), we can obtain the scale-dependent HI bias, which can be expressed as,

$$b_{\text{HI}}^{\text{NG}}(z, k) = b_{\text{HI}}(z) + \Delta b_{\text{HI}}(z, k), \quad (14)$$

in which  $b_{\text{HI}}^{\text{NG}}(z, k)$  is the total bias,  $b_{\text{HI}}(z)$  is the scale-independent term, and  $\Delta b_{\text{HI}}(z, k)$  is the scale-dependent term, which is obtained by integrating  $\Delta b(M, z, k)$  over the halo mass function and HI mass model,

$$\Delta b_{\text{HI}}(z, k) = \frac{1}{\rho_{\text{HI}}(z)} \int_{M_{\text{min}}}^{M_{\text{max}}} dM \times \frac{dn}{dM}(M, z) M_{\text{HI}}(M) \Delta b(M, z, k), \quad (15)$$

where  $\rho_{\text{HI}}(z)$  is calculated in Eq. (9).

Dalal et al. [12] firstly derived the expression of scale-dependent correction to the bias of galaxies and halos for local-shape bispectrum,

$$\Delta b^{\text{D}}(z, k) = 2(b_{\text{E}} - 1) f_{\text{NL}} \delta_{\text{c}} \frac{3\Omega_{\text{m}}}{2a(z)g(z)r_H^2 k^2}, \quad (16)$$

in which,  $\delta_{\text{c}}$  is the critical density,  $a(z)g(z) = D(z)$  is the linear growth factor and  $r_H = 1/H_0$ . Equation (16) is derived by only considering the high peaks of the density contrast, which means that the expression only works at the large scales with  $k \rightarrow 0$ .

More accurate analytical expressions for the scale-dependent bias have been studied [13–18]. A widely used expression is derived by Matarrese and Verde [13],

$$\Delta b^{\text{MV}}(M, z, k) = 2f_{\text{NL}} \left( \frac{\delta_{\text{c}}^2(z)}{\sigma_R^2} \right) \frac{\mathcal{F}(k)}{\mathcal{M}_R(k)}, \quad (17)$$

in which,  $\delta_{\text{c}}(z) = \delta_{\text{c}}/D(z)$ <sup>1</sup> and  $\mathcal{M}_R(k)$  is the Eq. (2) smoothed with window function  $W_R(k)$ ,

$$\mathcal{M}_R(k) = \frac{2}{3} \frac{T(k)k^2}{H_0^2 \Omega_{\text{m}}} W_R(k), \quad (18)$$

where  $R$  denotes a smoothing radius which defines the halo mass  $M$  by,

$$M = \frac{3H_0^2 \Omega_{\text{m}}}{8\pi G} \frac{4\pi R^3}{3}. \quad (19)$$

So  $\Delta b^{\text{MV}}$  is also a function of halo mass,  $M$ .  $\mathcal{F}(k)$  is related to the bispectrum of primordial potential field  $B_{\phi}(k_1, k_2, k)$ , and the power spectrum  $P_{\phi}(k)$ ,

$$\mathcal{F}(k) = \frac{1}{16\pi^2 \sigma_R^2} \int dk_1 k_1^2 \mathcal{M}_R(k_1) \times \int_{-1}^1 d\mu \mathcal{M}_R(k_2) \frac{B_{\phi}(k_1, k_2, k)}{P_{\phi}(k)}, \quad (20)$$

where  $k_2^2 = k^2 + k_1^2 + 2kk_1\mu$  and  $\sigma_R$  is the *rms* of the underlying dark matter fluctuation fields smoothed on scale  $R$  given in Eq. (19).

If we substitute the local-shape bispectrum into Eq. (20), and take the limit of  $k \rightarrow 0$ , then<sup>2</sup> the dependence of  $\Delta b^{\text{MV}}(M, z, k)$  on halo mass automatically drops off

$$\begin{aligned} \mathcal{F}(k \rightarrow 0) &\rightarrow 1 \\ T(k \rightarrow 0) &\rightarrow 1 \\ \mathcal{M}_R(k \rightarrow 0) &\rightarrow (2/3)k^2/(H_0^2 \Omega_{\text{m}}), \end{aligned}$$

<sup>1</sup> This is consistent with eq. 13 in [13]. The “ $\Delta_{\text{c}}$ ” defined in [13] is equal to  $\delta_{\text{c}}$  in this paper.

<sup>2</sup> In Ref. [13]  $b_{\text{E}} - 1 = b_{\text{L}} = \delta_{\text{c}}/\sigma_R^2$



and,

$$\begin{aligned}\Delta b^{\text{MV}}(z, k \rightarrow 0) &\rightarrow 2(b_E - 1)f_{\text{NL}} \frac{\delta_c}{a(z)g(z)} \frac{3}{2} \frac{H_0^2 \Omega_m}{k^2} \\ &= \Delta b^{\text{D}}(z, k) \\ &\sim k^{-2},\end{aligned}\quad (21)$$

i.e. the general expression of scale-dependent bias in Eq. (17) recovers the bias proposed in Dalal et al. [12]. The advantage of using Eq. (17) is that it can be used to calculate any shape of PNG, provided that the bispectrum  $B_\phi$  function is given.

The scale-dependent bias for equilateral, orthogonal and enfolded shapes of PNG can be obtained by substituting Eqs. (4), (6) and (7) into Eq. (20). In Fig. 2, we show the absolute value of the scale-dependent part of the bias, i.e. Eq. (15) for the four shapes of PNG at  $z = 0$  (left panel) and  $z = 2$  (right panel). One can see that the local shape has most prominent feature of scale-dependent bias at large scales, which can be constrained with 21-cm intensity mapping observation on large angular scales. The orthogonal and enfolded shapes have less prominent features but possibly detectable at small  $k$ . The scale-dependent bias induced by equilateral shape is too small on large scales so it will be hard to be detected. The results shown in Fig. 2 are consistent with the analysis in [18] and fig. 1 in [46].

We can see the asymptotic behavior of scale-dependent bias (Eq. (15)) on large scales by taking the limit of  $k \rightarrow 0$ , then  $\Delta b \rightarrow (\mathcal{F}/\mathcal{M}_R)$ . Therefore,

$$\begin{aligned}\Delta b(\text{Local}) &\sim k^{-2} \\ \Delta b(\text{Equilateral}) &\sim \text{const} \\ \Delta b(\text{Enfolded}) &\sim k^{-1} \\ \Delta b(\text{Orthogonal}) &\sim k^{-1}.\end{aligned}\quad (22)$$

These asymptotic behaviors of  $\Delta b$  is consistent with the computation of halo models in Fig. 2.

### C. Power spectrum

We employ the HI tomographic angular power spectrum as the observable in our analysis, The expression of the angular power spectrum of the  $i$ -th and the  $j$ -th redshift bins is,

$$C_\ell^{ij} = 4\pi T_b^{ij} \int d \ln k \mathcal{W}_\ell^i(k) \mathcal{W}_\ell^j(k) \Delta_\zeta^2(k), \quad (23)$$

in which,  $\Delta_\zeta^2(k)$  is the dimensionless power spectrum of primordial curvature perturbation and  $T_b^{ij} = T_b(z_i)T_b(z_j)$  is the multiplication of HI mean brightness temperature of the  $i$ -th and  $j$ -th redshift bins. We use the expression of  $T_b(z)$  in Chang et al.(2008) [47],

$$\begin{aligned}T_b(z) &= 0.39 \left( \frac{\Omega_{\text{HI}}}{10^{-3}} \right) \left( \frac{1+z}{2.5} \right)^{0.5} \\ &\times \left( \frac{\Omega_m + (1+z)^{-3} \Omega_\Lambda}{0.29} \right)^{-0.5} \text{ mK},\end{aligned}\quad (24)$$

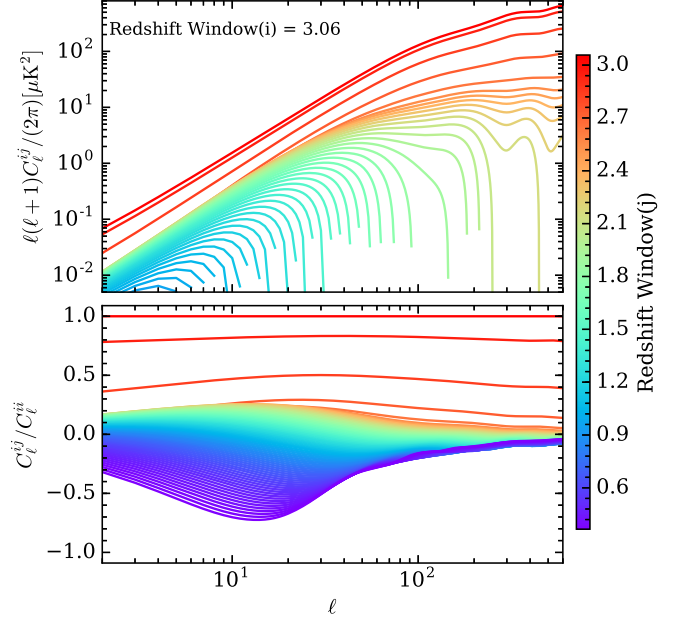


FIG. 3: *Upper panel:* Cross-correlated angular power spectrum between redshift  $z_i = 3.06$  and  $z_j$ , which ranges from 0.37 to 3.06 shown with different colors. *Lower panel:* The ratio of tomographic angular cross-power spectrum between  $z_i$  and  $z_j$  to the auto-power spectrum of  $z_i$ .

where  $\Omega_{\text{HI}}$  is the fractional HI density assumed to be  $0.62 \times 10^{-3}$  [48]. The window function  $\mathcal{W}_\ell(k)$  is,

$$\mathcal{W}_\ell(k) = \int d\chi \frac{dN_g(\chi)}{d\chi} j_\ell(k\chi) b_{\text{HI}}^{\text{NG}}(\chi(z), k) T_\delta(\chi, k), \quad (25)$$

where  $j_\ell$  is a spherical Bessel function,  $dN_g(\chi)/d\chi$  is the redshift distribution of galaxy number,  $T_\delta(\chi, k)$  is the transfer function for the galaxy number over-density, and  $b_{\text{HI}}^{\text{NG}}$  is the total bias of HI (Eq. (14)). To calculate the angular power spectrum, we use the CAMB\_SOURCES package [49].

Figure 3 shows the tomographic angular power spectrum. The *Upper panel* shows the cross-power spectrum between redshift  $z_i = 3.06$  and  $z_j$ , which ranges from 0.37 to 3.06 shown with different colors. The *Lower panel* shows the ratio of the cross-power spectrum of different redshift bins to the auto-power spectrum of the same redshift bin. We can see that the cross-power spectrum decreases as the redshift deviates from  $z_i = 3.06$ . This is what we expected, since the cross-correlated signal should drop if the frequency windows move away from each other.

### IV. FISHER MATRIX FORECAST

To forecast the potential for constraining  $f_{\text{NL}}$ , we perform the Fisher matrix analysis. If we assume that the model likelihood surface in parameter space can be well approximated by a multivariant Gaussian, the Fisher matrix  $\mathbf{F}$  is then a

good approximation for the inverse of the parameter covariance. In the 21-cm tomography, each frequency band will provide a map of 21-cm intensities, so we need to sum over the Fisher matrix in both  $\ell$ -space and frequency space. Since  $\nu = 1420\text{MHz}/(1+z)$ , each frequency corresponds to a unique redshift slice. The Fisher matrix is,

$$\mathbf{F}_{\alpha\beta} = f_{\text{sky}} \sum_{\ell_{\min}}^{\ell_{\max}} \left( \frac{2\ell+1}{2} \right) \text{tr}[\mathbf{C}_{\ell,\alpha} \mathbf{\Sigma}_{\ell} \mathbf{C}_{\ell,\beta} \mathbf{\Sigma}_{\ell}], \quad (26)$$

in which,  $\mathbf{C}_{\ell}$  is an  $n_z \times n_z$  matrix, in which each element is the HI cross angular power spectrum between the two frequency bins.  $\mathbf{\Sigma}_{\ell} = (\mathbf{C}_{\ell} + \mathbf{N}_{\ell})^{-1}$  is the *total* noise inverse matrix, in which  $\mathbf{N}_{\ell}$  is the  $n_z \times n_z$  experimental noise power spectrum. Here we make a simple assumption that the noises in different frequency (redshift) bins are uncorrelated, therefore the  $\mathbf{N}_{\ell}$  is a diagonal matrix. In reality, 21-cm intensity maps are highly contaminated by the foreground, such as Galactic synchrotron emission, extragalactic point sources, and atmospheric signal. One needs to apply foreground removal technique to reduce the foreground contamination [50–52]. However, there always be some level of residual Galactic foreground after applying such techniques to the maps. Therefore the cross-correlation of noises between different frequency bands may not completely be zero.

Under our simplified assumption, the element of  $\mathbf{N}_{\ell}$  matrix is

$$\begin{aligned} N_{\ell}^{ij} &= \delta^{ij} N_{\ell}^{\text{HI}} \\ &= \delta^{ij} T_{\text{sys}}^2 S_{\text{survey}} / (N_{\text{ant}} N_{\text{feed}} t_{\text{TOT}} \Delta\nu). \end{aligned} \quad (27)$$

$T_{\text{sys}} = T_{\text{rec}} + T_{\text{sky}}$  is the system temperature, which is contributed from the sky temperature,  $T_{\text{sky}} = 60 \times (300\text{MHz}/\nu)^{2.55}$ , and receiver temperature  $T_{\text{rec}}$  for each experiment.  $N_{\text{ant}}$  and  $N_{\text{feed}}$  are the number of antenna and the number of feed horn in each antenna respectively. The detailed experimental parameters for FAST, SKA-I and BINGO are list in Table I.

## V. EXPERIMENT PARAMETERS

TABLE I: The experiment parameters for FAST, SKA-I and BINGO.  $D_{\text{dish}}$  is the illuminated aperture.

	FAST	SKA-I	BINGO
$\nu_{\min}[\text{MHz}]$	1050	350	960
$\nu_{\max}[\text{MHz}]$	1350	1050	1260
$\Delta\nu[\text{MHz}]$	10	10	10
$n_{\nu}(n_z)$	30	70	30
$D_{\text{dish}}[\text{m}]$	300	15	25
$N_{\text{ant}} \times N_{\text{feed}}$	$1 \times 19$	$190 \times 1$	$1 \times 60$
$t_{\text{TOT}}[\text{yr}]$	1	1	1
$T_{\text{rec}}[\text{K}]$	25	28	50
$S_{\text{survey}}[\text{deg}^2]$	$< 24000$	$< 25000$	2500

*a. BINGO* The BINGO experiment is a single-dish HI intensity mapping experiment, which aims at mapping the HI emission at frequencies between 960MHz and 1260MHz [27, 53]. The telescope of BINGO experiment has no moving parts and it conducts a drift-scan strategy. To achieve enough survey area, a wide instantaneous field of view (FOV) with multiple feeds is required. A total of 60 feeds laid out in a rectangle of  $16\text{m} \times 15\text{m}$  at the focal plane. This will form a FOV of about  $10^\circ$  (in Declination direction)  $\times 9^\circ$  (in Right Ascension direction). With the  $10^\circ$  wide strip centering at Declination of  $-45^\circ$ , the total survey area is about  $2500 \text{deg}^2$ .

*b. FAST* FAST is the largest single-dish telescope, which also has the multi-beam system of 19 feed-horns array [28, 29]. The multi-beam system is proposed to work at frequencies from 1.05GHz to 1.45GHz with system temperature of 25K. In our analysis, we only include the frequencies up to 1.35GHz. With the 300m illuminated aperture, each of the feed-horn has the beam size (Full Width at Half Maximum) of  $2.9'$ , and form a  $26'$  FOV with 19 beams. Due to the long slewing time, FAST can only work on drift-scan observation mode. Similar to the BINGO experiment, FAST scans a  $26'$  wide strip along the Right Ascension direction for each sidereal day. But the zenith angle of FAST can be adjusted from Dec:  $-14^\circ 12'$  to Dec:  $65^\circ 48'$ . Without over lapping between scanning strips, it takes about half year to cover all  $80^\circ$  Declination range. With one-year observation ( $3.15 \times 10^7$  second), the maximum survey area is about  $24000 \text{deg}^2$ .

*c. SKA-I* The SKA Phase I (SKA-I) plans to construct 190 movable 15m dishes [26]. The maximum survey area is about  $25000 \text{deg}^2$ . A efficient survey area is need to be explored to minimal the constraint errors. In our analysis, we only consider the autocorrelation of each dishes, which means that the SKA-I works as 190 single dishes. Without the interferometry, the SKA-I has very low resolution and is only sensitive to the low- $\ell$  modes.

Figure 4 shows the noise power spectra of different experiments in at redshift bin  $z = 0.37$  (left upper panel) and  $z = 3.06$  (right upper panel). The black solid line in the upper panel of each figure shows the standard angular power spectrum of 21-cm ( $f_{\text{NL}} = 0$ ); The black dash-dotted, dotted and dashed lines show the noise power spectra of SKA-I, FAST and BINGO experiments. One-year observation time and  $2500 \text{deg}^2$  survey area are assumed for all the experiments. The partial derivatives of  $C_{\ell}^{ii}$  with respect to parameter  $f_{\text{NL}}$  are shown in the lower panel. The different colors correspond to different types of PNG.

Comparing to the BINGO experiment, FAST and SKA-I can have very large survey area. However, with the limit integration time, large survey area may not be able to beat down the constraint error. We will discuss the details in Sect. VI.

## VI. RESULTS AND DISCUSSION

Figure 5 shows the  $\sigma_{f_{\text{NL}}}$  contours for local-shape PNG in the plane of the survey area and total observation time. The left and middle panels of Fig. 5 show the contours for SKA-I

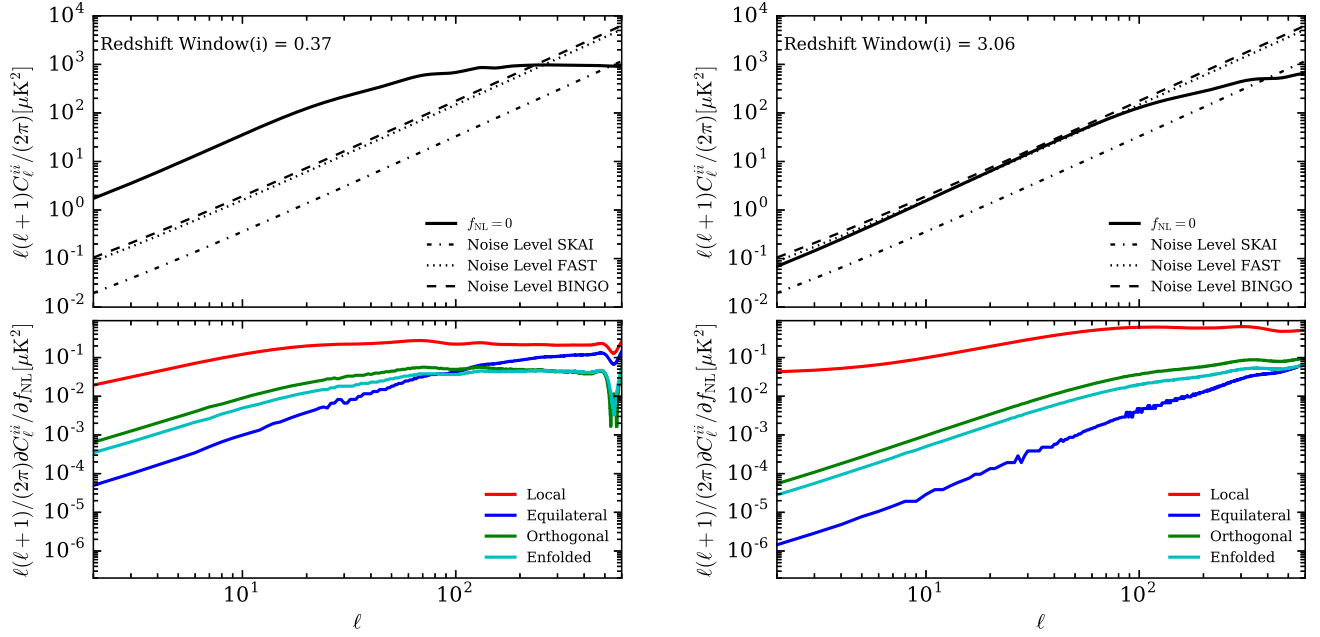


FIG. 4: *Upper panels*—Comparison between the noise power spectra of different experiments and the 21-cm power spectrum in standard model ( $f_{NL} = 0$ ) for the two representative redshift bins (left and right panels). In both panels, one-year observation time (equivalent to  $3.15 \times 10^7$  sec) and 2500 deg<sup>2</sup> survey area are assumed for all the experiments. *Lower panels*—The partial derivatives of  $C_{\ell}^{ii}$  with respect to parameter  $f_{NL}$  for four shapes of PNG.

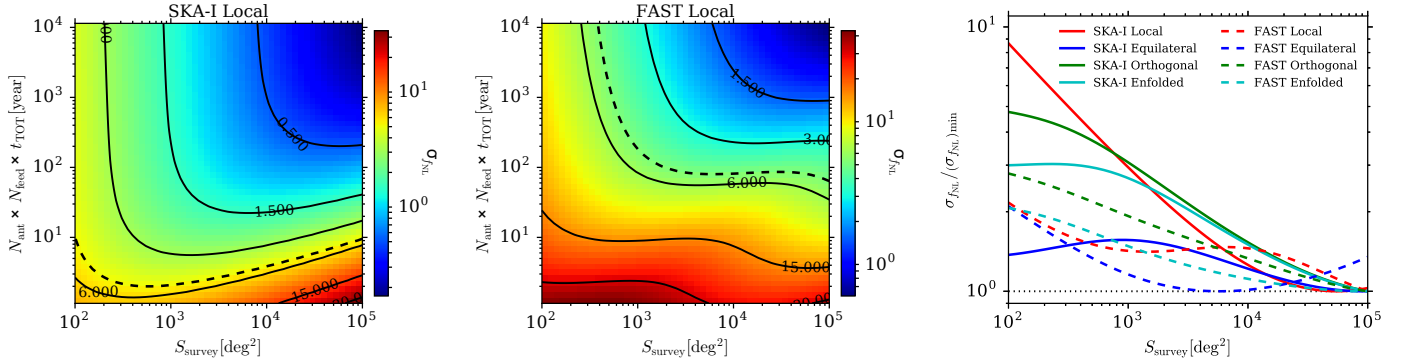


FIG. 5: The left (for SKA-I) and middle (for FAST) panels show the  $\sigma_{f_{NL}}$  contours for local-shape PNG in the parameter space of the survey area and total observation time. The dashed contour is the error of constraint with *Planck* temperature and polarization data [11]. The right panel shows the  $\sigma_{f_{NL}}/(\sigma_{f_{NL}})_{min}$  as a function of survey area for various PNG types. The solid lines show the results for SKA-I with one-year observational time and 190 dishes; the dashed lines show the results for FAST with one-year observational time and 19 beams.

and FAST experiments respectively. The colour going from red to blue means that the constraints become stronger. Different black solid lines are the contours of the same error of  $f_{NL}^{local}$ . Therefore, the error tends to become smaller if  $N_{ant} \times N_{feed} \times t_{TOT}$  becomes bigger. Thus the most efficient way to reduce the constraint error is to increase the observation time or the number of dishes(feeds). Assuming one-year observation time and the maximum dish(feeds) number for SKA-I and FAST experiments, the constraint errors of various PNG types as function of survey area are shown in the right panel of Fig. 5. In order to have a clear view, the constraint errors,  $\sigma_{f_{NL}}$ , are divided by the their minimal values. It is true that the optimal survey area may not be the maximal survey area. For example, in the case of Equilateral shape, the

optimization is about 6000 deg<sup>2</sup> for FAST experiment. For other shapes, the optimized survey areas are approaching to the maximum sky coverage of SKA-I or FAST. The large survey area can help to beat the cosmic variance on large scales, but the integration time per pixel becomes smaller, leading to larger pixel noise.

One can see from the right panel of Fig. 5 that generally speaking the larger survey area is, the smaller the error of  $f_{NL}$  is, except for measuring equilateral shape of PNG using FAST survey. This is different from the situation of using 21-cm intensity mapping to measure the angular scale of BAO acoustic oscillation, which have the optimal survey area around 6000 deg<sup>2</sup> (For BINGO see fig. 7 in [27], and for FAST see fig. 1 in [54]). The reason is because scale-dependent bias

from PNG is always prominent on very large scales, so beating down cosmic variance is more important than lowering down the pixel noise. However, BAO scale is sub-horizon for which one needs to find a balance between lowering down pixel noise and beating down cosmic variance. We use different optimized survey areas for different cases in the later analysis.

Figure 6 shows the  $\sigma_{f_{\text{NL}}}$  as a function of  $\ell_{\text{min}}$  if we fix  $\ell_{\text{max}} = 600$ . Different PNG shapes are shown in different panels. In each panel, different colors indicate different experiments as shown in the legends. The optimized survey areas are applied to the analysis. The constraint errors of different PNG shapes from *Planck* satellite are shown with the black dashed lines [11]. The  $\sigma_{f_{\text{NL}}}$  of different PNG shapes forecasted with different experiments are list in Table II.

We can see that, for the local shape PNG, the SKA-I experiment is potentially able to constrain  $f_{\text{NL}}$  better than *Planck* experiment. But we should realize that it is only the most ideal case. It is well known that, one of the big challenges for observations of HI intensity mapping is the foreground subtraction, and the low- $\ell$  modes may not be detectable due to the foreground contamination. Our results show that, to obtain a remarkable constraint on  $f_{\text{NL}}$  with the SKA-I intensity mapping in the future, we need to recover the angular power spectrum of HI with the minimal  $\ell_{\text{max}} \simeq 50$ . This is the aim of several recent efforts of restoring large angular power with cross-correlation with weak gravitational lensing [55, 56]. We also find that the constraint error for orthogonal shape PNG with SKA-I is  $\sim 25$ , which is at the same level of current *Planck* limit. If the observation can be extended to 2 years, the error will be reduced to  $\sim 20$ .

The constraint error for equilateral shape PNG with FAST is  $\sim 44$ , which is better than the results of SKA-I and BINGO experiments. The FAST error on  $f_{\text{NL}}^{\text{equil}}$  is close to the current limit of *Planck* experiment. This is because the scale-dependent bias induced by the equilateral shape PNG has higher signal-to-noise ratio at small scales and the FAST experiment is more sensitive to the small-scale modes than SKA-I single dish mode and BINGO.

We also test the possible extensions of the current configuration by adding more integration time. If the observation time for SKA-I and FAST could be extended to 2 years, the constraints on  $f_{\text{NL}}$  can be improved quantitatively. The forecasted constraint on different shapes of PNG are listed in Table II. It is worth noticing that the constraint error on orthogonal shape PNG with SKA-I and equilateral shape PNG with FAST become smaller than limits of *Planck* with extended observational time.

A good extension for FAST experiment is to extend its bandwidth to the lower frequencies, which are corresponding to the higher redshifts. So far the FAST telescope has one ultra-wide band receiver working on  $270\text{MHz} \sim 1.62\text{GHz}$ . Unfortunately, the ultra-wide band receiver has only one beam. It will take quite long time to achieve the same observation time as the multi-beam receiver. Now the multi-beam system of the FAST telescope is designed to work on frequencies between  $1050\text{MHz}$  and  $1350\text{MHz}$ . Assuming that the FAST multi-beam system works on the frequencies be-

tween  $350\text{MHz}$  and  $1050\text{MHz}$ , which is the same as the frequency range of SKA-I experiment, the constraint for local shape PNG will be  $\sigma_{f_{\text{NL}}^{\text{local}}} \sim 1.62$  with the optimized survey area of  $6000 \text{ deg}^2$ . The constraint errors ( $\sigma_{f_{\text{NL}}}$ ) for orthogonal and enfolded shapes become 39 and 64 respectively, which are all highly reduced.

## VII. CONCLUSION

In this work, we explored the constraining power on the primordial non-Gaussianity (PNG), with the future single-dish HI intensity mapping observations with BINGO, FAST and SKA-I. Four fundamental shapes of PNG are studied in our analysis, including local, equilateral, orthogonal and enfolded. We focus on the effect of scale-dependent bias to the underlying dark matter tracer, induced by the primordial non-Gaussianity. The properties of such scale-dependent bias at large-scale limit are discussed in our analysis. The forecast results are list in Table II.

Our forecasts show that with the current configuration of the experiments one-year observation time, the constraint on local shape of PNG from SKA-I intensity mapping experiment can be better than the current *Planck* experiment. The optimized survey area of  $25000 \text{ deg}^2$  is applied in the analysis of SKA-I, but the results are more sensitive to the total observation time than the survey area. However, the HI intensity mapping experiments may be contaminated by the foreground and the low- $\ell$  modes may be detectable. Our analysis shows that the SKA-I experiment can still have the remarkable constraint without the modes of  $\ell \lesssim 50$ . With two-years observation, the constraint on orthogonal shape PNG is  $\sim 20$ , which is also better than the constraint from *Planck* measurement.

The FAST experiment has its advantage of higher angular resolution, and more sensitive to the small-scale modes, which is good for constraining the equilateral shape of PNG. With the current configuration and two-years observation, the constraint error for Equilateral shape of PNG will be  $\sigma_{f_{\text{NL}}} = 32$ , which is better than the current limit of *Planck* observation.

Similar constraint on the local shape of PNG can be achieved by the FAST HI intensity mapping, if its frequency bandwidth could be extended to the lower frequencies (ultra-wide band). Assuming the same working frequency range, the best constraint from FAST on local shape of primordial non-Gaussianity is  $\sigma_{f_{\text{NL}}} \sim 1.62$ .

The studies we conduct here are the standard power spectra analysis of 21-cm. There have been efforts on using the multi-tracer technique to beat the cosmic variance and obtain tighter constraints on  $f_{\text{NL}}$  [57–59]. In addition, using three-point correlation function is another way to measure PNG. These methods will be explored to measure all shapes of  $f_{\text{NL}}$  in the future work.



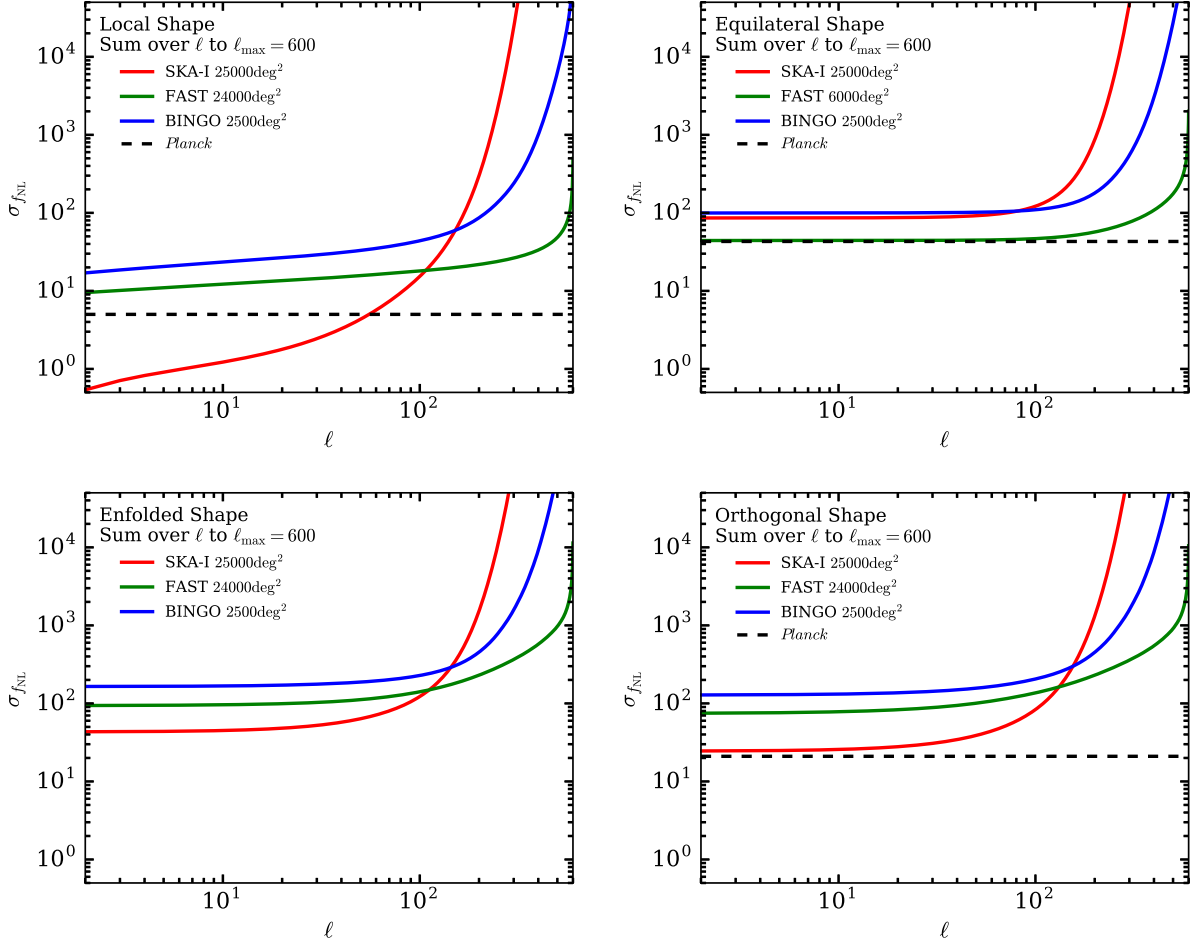


FIG. 6: The  $\sigma_{f_{\text{NL}}}$  as a function of  $\ell_{\text{min}}$  for various experiments and PNG shapes. The black dashed line is the current constraint with *Planck* temperature and polarization data [11].

TABLE II:  $\sigma_{f_{\text{NL}}}$  of different PNG shapes forecasted with different experiments. The optimized survey areas are applied in the analysis. The “*Planck* 2015” column shows the constraint error with *Planck* temperature and polarization data [11]. The numbers in bold character are the constraints better than *Planck*.

	<i>Planck</i> 2015	Current Configuration			Extentions		
		FAST	SKA-I	BINGO	SKA-I 2yr <sup>†</sup>	FAST 2yr <sup>††</sup>	FAST low <sup>‡</sup>
Local	5	9.5	<b>0.54</b>	17	<b>0.43</b>	7.4	<b>1.6</b>
Equilateral	43	44	86	100	66	<b>32</b>	53
Orthogonal	21	75	25	128	<b>20</b>	59	39
Enfolded	–	94	43	164	36	70	64

<sup>†</sup> SKA-I with two-year observation; <sup>††</sup> FAST with two-year observation; <sup>‡</sup> FAST with low frequencies range from 350MHz to 1050MHz

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